NONSTATIONARY HEAT TRANSFER ASSOCIATED WITH BOREHOLE WASHING

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A solution is obtained to the nonstationary axisymmetric problem of the temperature changes in a liquid which is injected into the borehole through a central (drill or delivery) pipe lowered to the bottom of the bore and expelled to the surface through the annular clearance about the pipe [1, 2].

The borehole, of constant depth L, is drilled in a soil with a natural steady temperature distribution and, prior to injection, is filled with a liquid whose temperature varies in proportion to the geothermal gradient. The liquid is pumped into the central tube at a constant rate Q at a known time-variable temperature. Two example cases are examined. In the first, the liquid enters the pipe at a constant temperature. In the second case, the temperature of the liquid injected through the central pipe is variable and equal to that of the liquid expelled through the annular clearance (closed-cycle circulation).

Both problems are of interest in drilling technology and in the evaluation of thermal effects on the oil bed.

1. We direct the Oz-axis of a cylindrical coordinate system with origin at ground level along the axis of the central pipe.

The heat flux equation of the liquid moving in the central pipe and in the annular clearance about it has the form

for $t > 0,\ 0 < z < L,\ 0 < r < R_1$

$$\pi R_1^2 \frac{\partial T_1}{\partial t} + \pi R_1^2 w_1 \frac{\partial T_1}{\partial z} = \frac{k}{c\rho} 2\pi R_1 (T_2 - T_1);$$

for $t > 0,\ 0 < z < L,\ R_1 < r < R$

$$\pi (R^2 - R_1^2) \frac{\partial T_2}{\partial t} - \pi (R^2 - R_1^2) w_2 \frac{\partial T_2}{\partial z} = \frac{k}{c\rho} 2\pi R_1 (T_1 - T_2) + q_w. \qquad (1.1)$$

Here T_1 and T_2 are the core temperatures of the fluid flowing in the central and annular pipes, respectively (it is assumed that the temperature in the cross section of each tube varies insignificantly with respect to these quantities and that the latter define the temperature of the liquid over the entire cross section); w_1 and w_2 are the mean flow rates of the liquid in the central and annular pipes, respectively (their values are equal to the flow rate divided by the corresponding area); q_w is the heat flow to the inner surface of the borehole; R_1 and R are the radii of the central pipe and the borehole, respectively; ρ and c are the density and heat capacity of the fluid; and k is the coefficient of heat transfer between the liquid in the central pipe and the liquid in the annular clearance.

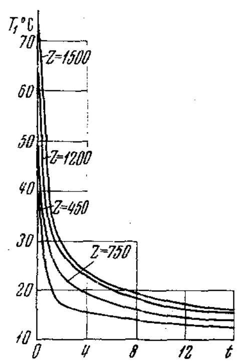

Fig. 1

The heat transfer coefficient depends on the flow rate, the thermophysical properties of the liquid, and geometry of the flow area, and is calculated from the known formula in heat transfer theory.

It can readily be seen that Eq. (1.1) does not contain terms which account for changes in the heat flow owing to the heat conduction along the axis and to the heat released in the fluid due to viscous friction. These terms are negligible in the first approximation as compared to the other terms in the equation.

Furthermore, it is assumed that the heat flows to the outer and inner surfaces of the central pipe are proportional to the temperature difference of the flow cores in the central and annular pipes.

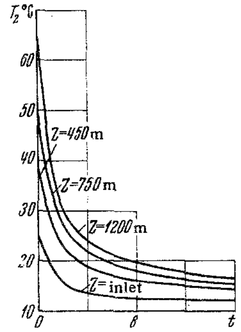

Fig. 2

The last equation of system (1.1) contains the value of the heat flow from the liquid to the inner surface of the heat flow from the liquid to the inner surface of the borehole. This value can be calculated by neglecting the heat transfer in the material of the borehole and by equating the heat flows and temperatures of the liquid and the soil at its surface,

$$q_w = 2\pi R\varkappa\lambda \left(\frac{\partial T_3}{\partial r}\right)_{r=R}, \quad T_2 = (T_3)_{r=R}, \quad \lambda = \frac{\lambda_3}{\lambda_1}. \tag{1.2}$$

Here λ is the ratio of the thermal conductivities of the soil and liquid; T_3 is the temperature of the soil, and $\varkappa$ is the thermal diffusivity of the liquid.

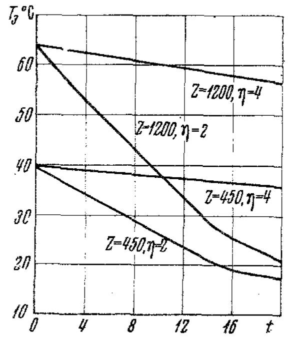

Fig. 3

For practically important time intervals the soil is heated along the radius over a length that is much smaller than that of the borehole, the temperature gradients along the radius and along the length of the borehole being roughly equal. Because of this, the term $\partial^2 T/\partial z^2$ in the heat flux equation may be neglected in comparison with the other terms [2]. The heat flux equation for soil has the following form:

$$\frac{\partial T_3}{\partial t} = \varkappa_3 \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_3}{\partial r}\right) \quad (t > 0,\ 0 < z < L,\ R < r < \infty),$$
$$T_3 = T_3(t, z, r), \tag{1.3}$$

where $\varkappa_3$ is the thermal diffusivity of the soil.

For the case in which the liquid enters the central pipe at a known variable temperature, the initial and boundary conditions may be written in the form

$$T_1 = T_2 = T_3 = T_* + \Gamma z \quad (t = 0,\ 0 \leqslant z \leqslant L,\ 0 \leqslant r < \infty),$$
$$T_1 = T_0(t) \quad (t > 0,\ z = 0), \tag{1.4}$$

$$T_1 = T_2 \quad (t > 0,\ z = L),$$
$$T_3 \to T_* + \Gamma z \quad (t > 0,\ r \to \infty),$$

where T_* is the temperature of the neutral layer; Γ is the geothermal gradient; and $T_0(t)$ is the temperature of the injected liquid.

We introduce the dimensionless variables

$$\omega = \frac{T - T_*}{T_*}, \quad \xi = \frac{z}{L}, \quad \eta = \frac{r}{R}, \quad \tau = \frac{w_1}{L} t \tag{1.5}$$

With these variables, Eqs. (1.1) and (1.3) and the limiting conditions (1.4) have the form

$$\begin{gathered}
\text{for } \tau > 0,\ 0 < \xi < 1,\ 0 < \eta < R_1/R \\
\frac{\partial \omega_1}{\partial \tau} + \frac{\partial \omega_1}{\partial \xi} = a(\omega_2 - \omega_1) \quad \left(a = \frac{kL2\pi R_1}{\rho c Q}\right), \\
\text{for } \tau > 0,\ 0 < \xi < 1,\ R_1/R < \eta < 1 \\
\frac{w_1}{w_2}\frac{\partial \omega_2}{\partial \tau} - \frac{\partial \omega_2}{\partial \xi} = a(\omega_1 - \omega_2) + \lambda b \left(\frac{\partial \omega_3}{\partial \eta}\right)_{\eta=1} \quad \left(b = \frac{2\pi\varkappa L}{Q}\right), \\
\text{for } \tau > 0,\ 0 < \eta < \infty,\ 0 < \xi < 1, \\
\frac{\partial \omega_3}{\partial \tau} = \frac{\alpha}{\eta}\frac{\partial}{\partial \eta}\left(\eta \frac{\partial \omega_3}{\partial \eta}\right) \quad \left(\alpha = \frac{\varkappa_3 \pi R_1^2 L}{R^2 Q}\right),
\end{gathered} \tag{1.6}$$

$$\begin{gathered}
\text{for } \tau = 0,\ 0 \leqslant \xi \leqslant 1,\ 0 \leqslant \eta < \infty \\
\omega_1 = \omega_2 = \omega_3 = \gamma\xi \quad (\gamma = \Gamma L / T_*), \\
\omega_1 = \omega_0(\tau) \quad (\tau > 0,\ \xi = 0), \\
\omega_1 = \omega_2 \quad (\tau > 0,\ \xi = 1), \\
\omega_2 = (\omega_3)_{\eta=1} \quad (\tau > 0,\ 0 \leqslant \xi \leqslant 1), \\
\omega_3 \to \gamma\xi \quad (\tau > 0,\ \eta \to \infty).
\end{gathered} \tag{1.7}$$

We introduce the new function

$$\theta = \omega - \gamma\xi. \tag{1.8}$$

For this function, Eqs. (1.6) and conditions (1.7) have the form

$$\begin{gathered}
\frac{\partial \theta_1}{\partial \tau} + \frac{\partial \theta_1}{\partial \xi} + \gamma = a(\theta_2 - \theta_1), \quad \theta_1 = \theta_1(\tau, \xi), \\
\frac{w_1}{w_2}\frac{\partial \theta_2}{\partial \tau} - \frac{\partial \theta_2}{\partial \xi} - \gamma = a(\theta_1 - \theta_2) + \lambda b \left(\frac{\partial \theta_3}{\partial \eta}\right)_{\eta=1}, \quad \theta_2 = \theta_2(\tau, \xi), \\
\frac{\partial \theta_3}{\partial \tau} = \frac{\alpha}{\eta}\frac{\partial}{\partial \eta}\left(\eta \frac{\partial \theta_3}{\partial \eta}\right), \quad \theta_3 = \theta_3(\tau, \xi, \eta),
\end{gathered} \tag{1.9}$$

$$\begin{gathered}
\theta_1 = \theta_2 = \theta_3 = 0 \quad (\tau = 0,\ 0 \leqslant \xi \leqslant 1,\ 0 \leqslant \eta < \infty), \\
\theta_1 = \omega_0 \quad (\tau)\ (\tau > 0,\ \xi = 0), \\
\theta_1 = \theta_2 \quad (\tau > 0,\ \xi = 1), \\
\theta_2 = (\theta_3)_{\eta=1} \quad (\tau > 0,\ 0 \leqslant \xi \leqslant 1), \\
\theta_3 \to 0 \quad (\tau > 0,\ \eta \to \infty).
\end{gathered} \tag{1.10}$$

2. We apply the one-dimensional Laplace-Karlson transformation with respect to the variable τ to Eqs. (1.9) and conditions (1.10). The transform of the function θ, which results from transformation with respect to the variable τ, we term Θ, i.e.,

$$\Theta(p, \xi, \eta) = p\int_0^\infty e^{-p\tau}\theta(\tau, \xi, \eta)\, d\tau. \tag{2.1}$$

As a result of the transformation, we obtain

$$\begin{gathered}
p\Theta_1 + \frac{d\Theta_1}{d\xi} + \gamma = a(\Theta_2 - \Theta_1), \quad \Theta_1 = \Theta_1(p, \xi), \quad \Theta_2 = \Theta_2(p, \xi), \\
up\Theta_2 - \frac{d\Theta_2}{d\xi} - \gamma = a(\Theta_1 - \Theta_2) + \lambda b \left(\frac{\partial \Theta_3}{\partial \eta}\right)_{\eta=1} \quad \left(u = \frac{w_1}{w_2}\right), \\
p\Theta_3 = \frac{\alpha}{\eta}\frac{\partial}{\partial \eta}\left(\eta \frac{\partial \Theta_3}{\partial \eta}\right), \quad \Theta_3 = \Theta_3(p, \xi, \eta),
\end{gathered} \tag{2.2}$$

$$\begin{gathered}
\Theta_1 = \Omega_0 \quad (\xi = 0), \qquad \Theta_1 = \Theta_2 \quad (\xi = 1), \\
\Theta_3 \to 0 \quad (\eta \to \infty), \qquad \Theta_2 = (\Theta_3)_{\eta=1} \quad (0 \leqslant \xi \leqslant 1),
\end{gathered} \tag{2.3}$$

where Ω_0 is the transform of function $\omega_0(\tau)$.

The solution of the last equation in (2.2), which satisfies the condition at infinity, has the form

$$\Theta_3 = \Phi(p, \xi) K_0'(\sqrt{p/\alpha}\eta), \tag{2.4}$$

where $\Phi(p, \xi)$ is an unknown function of p and ξ determined from the condition for $\eta = 1$. It can be readily seen that the following chain of equalities is valid:

$$\left(\frac{\partial\Theta_3}{\partial\eta}\right)_{\eta=1} = -\Phi(p, \xi)\sqrt{p/\alpha} K_1(\sqrt{p/\alpha}) =$$
$$= -\sqrt{p/\alpha} f(p, \alpha)(\Theta_3)_{\eta=1} = -\sqrt{p/\alpha} f(p, \alpha)\Theta_2,$$
$$f(p, \alpha) = \frac{K_1(\sqrt{p/\alpha})}{K_0(\sqrt{p/\alpha})}. \tag{2.5}$$

where $K_0(x)$ and $K_1(x)$ are MacDonald functions with the subscripts zero and unity.

We substitute the last link of the chain into the second equation of system (2.2). As a result, we obtain a system of equations for determining the temperature of the liquid in the central and annular pipes:

$$p\Theta_1 + \frac{d\Theta_1}{d\xi} + \gamma = a(\Theta_2 - \Theta_1), \quad \Theta_1 = \Theta_1(p, \xi), \quad \Theta_2 = \Theta_2(p, \xi), \tag{2.6}$$
$$up\Theta_2 - \frac{d\Theta_2}{d\xi} - \gamma = a(\Theta_1 - \Theta_2) - \lambda b\sqrt{p/\alpha} f(p, \alpha)\Theta_2,$$
$$\Theta_1 = \Omega_0 \quad (\xi = 0), \qquad \Theta_1 = \Theta_2 \quad (\xi = 1). \tag{2.7}$$

In addition, from equalities (2.4) and (2.5), we obtain the following formula for determining the transform of the temperature in the soil

$$\Theta_3 = \Theta_2 \frac{K_0(\sqrt{p/\alpha}\eta)}{K_0(\sqrt{p/\alpha})}. \tag{2.8}$$

It is not difficult to obtain a solution of system (2.6), and then to determine the transform of the temperature in the soil Θ_3 from formula (2.8). This solution has the following form:

$$\Theta_1 = B_1 e^{\Lambda_1\xi} + B_2 e^{\Lambda_2\xi} + A,$$
$$\Theta_2 = [a^{-1}(\Lambda_1 + p) + 1] B_1 e^{\Lambda_1\xi} + [a^{-1}(\Lambda_2 + p) + 1] B_2 e^{\Lambda_2\xi} + A(1 + p/a) + \gamma/a, \tag{2.9}$$
$$A = -\frac{up\gamma + M\sqrt{p} f(p, \alpha)\gamma}{p(u+1)a + up^2 + M\sqrt{p} f(p, \alpha)(a + p)}, \quad M = \lambda b/\sqrt{\alpha},$$
$$\Lambda_{1,2} = {}^1/_2\,[p(u-1) + M\sqrt{p} f(p,\alpha)] \pm \{{}^1/_4\,[p(u-1) + M\sqrt{p} f(p,\alpha)]^2 +$$
$$+ p(u+1)a + up^2 + M\sqrt{p} f(p, \alpha)(a + p)\}^{1/2}.$$

If the liquid is injected into the central pipe at a known variable temperature, the constants of integration B_1 and B_2, determined from the boundary conditions, have the form

$$B_1 = -\frac{Ap + \gamma - (\Omega_0 + A)(\Lambda_2 + p)e^{\Lambda_2}}{e^{\Lambda_1}(\Lambda_1 + p) - e^{\Lambda_2}(\Lambda_2 + p)}, \qquad B_2 = -B_1 - \Omega_0 - A.$$

In the case of closed-cycle circulation, when the temperature of the liquid flowing out of the annular clearance is equal to that of the injected liquid, the constants B_1 and B_2 have the form

$$B_1 = \frac{(\gamma + pA)(e^{\Lambda_2} - 1)}{(\Lambda_1 + p)(e^{\Lambda_1} - e^{\Lambda_2})} \qquad B_2 = \frac{(\gamma + pA)(e^{\Lambda_1} - 1)}{(\Lambda_2 + p)(e^{\Lambda_2} - e^{\Lambda_1})}.$$

Inversion of the solutions obtained was performed by the numerical method of [3], by which inverse transform is sought as a series in Legendre polynomials:

$$\theta = \sum_{n=0}^{\infty} c_n P_{2n}(X), \quad X = e^{-\sigma\tau}. \tag{2.10}$$

In order to determine the series coefficients, one must know the values of the transform Θ at the equidistant points $p = (2n + 1)\sigma$.

Here σ is a positive number, and n = 0, 1, 2,. . . . The selection of the value for σ depends on the time interval within which the value of the inverse transform of θ must be computed.

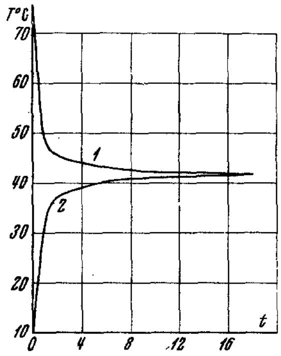

Fig. 4

The coefficients c_n are determined from a system of algebraic equations given in [3]. Making use of this method and of the transforms Θ_1, Θ_2, and Θ_3 obtained, we calculated the temperature of the liquid at several points of the borehole and soil for various times, both for constant temperature of the injected liquid and for closed-circuit circulation.

The calculations were performed for the following values of the quantities which define the heat transfer conditions:

$$c = 1 \text{ kcal/kg-}^\circ\text{C} \quad \rho = 10^3 \text{ kg/m}^3 \quad \lambda_1 = 0.53 \text{ kcal/m-}^\circ\text{C-hr},$$

$$\varkappa_3 = 2 \cdot 10^{-3} \text{ m}^2 \text{ hr} \quad \lambda_2 = 1 \text{ Kcal/m-}^\circ\text{C-hr} \quad Q = 30 \text{ liter/sec},$$

$$L = 1500 \text{ m}, \quad R_1 = 2.5'', \quad R = 4''.$$

The coefficient k of heat transfer between the liquid moving in the central pipe and that in the annular clearance was assumed to be constant and equal to 10^3 kcal/m$^2 \cdot {}^\circ$ C·hr. In the first case under consideration, we assumed that $T_0 = 9^\circ$ C, $\Gamma = 0.032^\circ$ C/m, and $T_* = 25^\circ$ C, and in the second case that $\Gamma = 0.041^\circ$ C/m and $T_* = 11.5^\circ$ C. The value of σ was set as 0.02.

The selection of the values in (2.11) was not arbitrary, but corresponded to the conditions of an experiment aimed at determining the cooling of the bottom of the borehole. In this experiment, the temperature of the liquid was measured at the inlet and at the bottom of the borehole into which a liquid with a temperature $T_0 = 9^\circ$ C was injected through the central pipe.

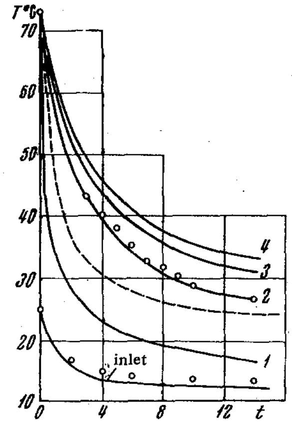

Fig. 5

For the injection of a liquid of constant temperature T_0, the results of numerical inversion with subsequent transition to dimensional values were used as a basis to plot the temperature of the liquid (T° C) against the time (t, in hr) at various points of the central pipe (Fig. 1) and the annular clearance (Fig. 2). The distance of a point from the borehole inlet (in m) is indicated at each curve.

Figure 3 shows a plot of the soil temperature against time (T_0 = const), from which it may be seen that heating of the soil is very slow (over a thickness of roughly four radii in the course of 20 hr).

For the case of closed-circuit circulation, the time-variation of the liquid at the bottom 1 and the inlet 2 of the borehole is shown in Fig. 4. The curves reveal the rapid temperature compensation in the circulating liquid.

In Fig. 5, the temperature of the liquid at the bottom and inlet of the borehole is plotted against time for various values of the heat transfer coefficient k. The curves numbered 1, 2, 3, and 4 correspond to the following values of k: 10^3, $2 \cdot 10^3$, $3 \cdot 10^3$, and $4 \cdot 10^3$ kcal/m$^2 \cdot$°C$\cdot$hr.

The plots obtained indicate that the heat transfer coefficient has a greater effect on the temperature of the liquid at the bottom than at the inlet of the borehole (the temperature curves at the inlet almost overlap). In addition, Fig. 5 shows (dashed line) the temperature variation curve of the liquid at the bottom of the borehole, plotted from the formulas proposed in [2] for a heat transfer coefficient $k = 2 \cdot 10^3$ kcal/m$^2 \cdot$° C$\cdot$hr. The circles in Fig. 5 refer to the temperatures of the liquid at the inlet and the bottom of the borehole that were measured in the experiment mentioned above. The experimental and theoretical results for $k = 2 \cdot 10^3$ kcal/m$^2 \cdot$° C$\cdot$hr are in good agreement.

The results of the analysis show that, owing to the substantial variations of the liquid temperature over large time intervals, one may not use the assumption of a stationary nature of the heat transfer [1] in the solution of the problem under consideration.

The experimental and theoretical results are in satisfactory agreement even for a constant value of the heat transfer coefficient.

The curves indicate that the formulas in [2], obtained by the method of successive steady-state shifts are well suited for calculating the temperature of the liquid. For the value of σ selected, the number of terms retained in the series (2.10) to ensure the accuracy desired varied from four to six, depending on the value of ξ.

Calculation of the series coefficients from the formulas proposed in [3] does not involve any major difficulties, and can be performed on a Rheinmetall computer.

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